Review

Dynamic Programming

Formulating dynamic programs – two ways

- A **dynamic program** models situations where decisions are made in a sequential process in order to optimize some objective
- **Stages** $t = 1, 2, \dots, T$
  - stage $T \leftrightarrow$ end of decision process
- **States** $n = 0, 1, \dots, N \leftarrow$ possible conditions of the system at each stage
- Two representations: **shortest/longest path** and **recursive**

Shortest/longest path		Recursive
node t_n	$\leftrightarrow$	state n at stage t
edge $(t_n, (t+1)_m)$	$\leftrightarrow$	allowable decision x_t in state n at stage t that results in being in state m at stage $t+1$
length of edge $(t_n, (t+1)_m)$	$\leftrightarrow$	cost/reward of decision x_t in state n at stage t that results in being in state m at stage $t+1$
length of shortest/longest path from node t_n to end node	$\leftrightarrow$	cost/reward-to-go function $f_t(n)$
length of edges (T_n, end)	$\leftrightarrow$	boundary conditions $f_T(n)$
shortest or longest path	$\leftrightarrow$	recursion is min or max: $f_t(n) = \min_{x_t \text{ allowable}} \text{ or } \max \left\{ \left(\begin{array}{c} \text{cost/reward of} \\ \text{decision } x_t \end{array} \right) + f_{t+1} \left(\begin{array}{c} \text{new state} \\ \text{resulting} \\ \text{from } x_t \end{array} \right) \right\}$
source node 1_n	$\leftrightarrow$	desired cost-to-go function value $f_1(n)$

- Note that the length of edge (T_n, end) is often 0, but not always!

Example 1. Simplexville Oil needs to build capacity to refine 1,000 barrels of oil and 2,000 barrels of gasoline per day. Simplexville can build a refinery at 2 locations. The cost of building a refinery is as follows:

Oil capacity per day	Gas capacity per day	Building cost ($ millions)
0	0	0
1000	0	5
0	1000	7
1000	1000	14

The problem is to determine how much capacity should be built at each location in order to minimize the total building cost. To make things a little simpler, assume that the capacity requirements must be met exactly.

a. Formulate this problem as a dynamic program by giving its shortest path representation.
b. Formulate this problem as a dynamic program by giving its recursive representation. Solve the dynamic program.